EXERCISE – IV HINTS & SOLUTIONS

Sol.1 $f'(x)$ is a quadratic

& $f'(-1) = 0$ and $f'\left(\frac{1}{3}\right) = 0$

so Let $f'(x) = a(x + 1)\left(x - \frac{1}{3}\right)$

$\Rightarrow f(x) = a\left(\frac{x^3}{3} + \frac{x^2}{3} - \frac{x}{3}\right) + b$

(b = 2 at x = –2)
y = 0

$$\frac{a}{3}\int_{-1}^{1}(x^3 + x^2 - x + 2)dx = \frac{14}{3}$$

$\Rightarrow$ a = 3 so $f(x) = x^3 + x^2 - x + 2$

Sol.2 (a) $y = x + \sin 2x$

$\frac{dy}{dx} = 1 + 2\cos 2x$

$\frac{dy}{dx} = 0 \Rightarrow \cos 2x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

$f(0) = 0$

$f(2\pi) = 2\pi$

$f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

$f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

max. value = 2π

min. value = 0

(b) $y = 2\cos 2x - \cos 4x$

$\frac{dy}{dx} = -4\sin 2x + 4\sin 4x = 0$

$\Rightarrow x = 0, \frac{\pi}{2}, \frac{\pi}{6}$

$f(0) = 1$

$f(\pi) = 1$

$f\left(\frac{\pi}{2}\right) = -3$

$f\left(\frac{\pi}{6}\right) = \frac{3}{2}$

min. value = -3

max. value = $\frac{3}{2}$

Sol.3 The value of the determinent is

$$D = \frac{f(x)}{x^3} + 1$$

$$\lim_{x\to 0}\frac{1}{x}\ell n\left(\frac{f(x)}{x^3}+1\right) = \lim_{x\to 0}\ell n\left(\frac{f(x)}{x^3}+1\right)^{1/x}$$

= 2 (given)

For the existance of limit coefficient of x^3, x^2, x & constant term of f(x) is zero.

Now $\ell n\, e^{\lim_{x\to 0}\left(\frac{f(x)}{x^3}\right)\cdot\frac{1}{x}} = \lim_{x\to 0}\frac{ax^6 + bx^5 + cx^4}{x^4} = 2$

$\Rightarrow$ c = 2

$f'(x) = x^3(6ax^2 + 5bx + 8)$

$f'(1) = 0$ & $f'(2) = 0 \Rightarrow 6a + 5b + 8 = 0$

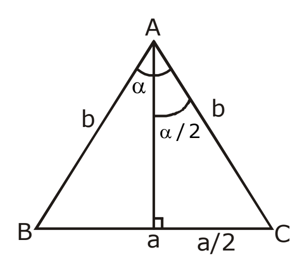
& $24a + 10b + 8 = 0$

on solving $a = \frac{2}{3}$, $b = -\frac{12}{5}$

so $f(x) = \frac{2}{3}x^6 - \frac{12}{5}x^5 + 2x^4$

Sol.4 Perimeter will be maximum when both the other sides are of same length

$\sin \alpha/2 = \frac{a/2}{b}$



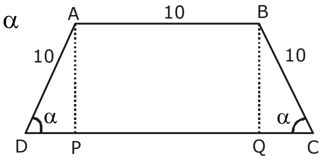
$b = \frac{a}{2}\text{cosec}\,\frac{\alpha}{2}$

Perimeter = a + 2b

$= a + a\,\text{cosec}\,\frac{\alpha}{2}$

Sol.5 AP = BQ = 10 sin α

DP = QC = 10 cos α



$$\text{Area} = \frac{1}{2}(AB+CQ)\times BQ$$

$$= \frac{1}{2}\,(10 + 10 + 20\cos\alpha)\; 10\sin\alpha$$

$$A = 100\left(\sin\alpha + \frac{1}{2}\sin 2\alpha\right)$$

$$\frac{dA}{d\alpha} = 100\,(\cos\alpha + \cos 2\alpha) = 0$$

$\cos 2\alpha = -\cos\alpha = \cos(\pi - \alpha)$

$\alpha = \pi/3$

$$\frac{d^2A}{d\alpha^2} = 100\,(-\sin\alpha - 2\sin 2\alpha)\Big|_{\alpha=\frac{\pi}{3}} < 0$$

$$\text{So } A_{max} = 100\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}\right]$$

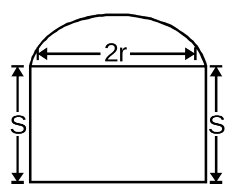
$$= 75\sqrt{3} \text{ Sq. units}$$

Sol.6 Area of the pool

$$A = \frac{\pi r^2}{2} + (2r)\,.\,s \qquad .....(i)$$

perimeter :

P = 25 + 2r + πr .....(ii)



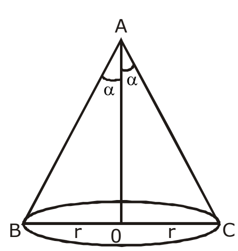
from (i) $\frac{A}{r} = \frac{\pi r}{2} + 25$

so $P = \frac{A}{r} + 2r + \frac{\pi r}{2}$

$$\frac{dp}{dr} = 0 \Rightarrow r = \sqrt{\frac{2A}{\pi+4}} \;\&\; s = \sqrt{\frac{2A}{\pi+4}}$$

Sol.7 $S = \pi r\,(r + \ell)$ = constant

$$v = \frac{1}{3}\pi r^2 h$$



$$z = v^2 = \frac{1}{9}\pi^2 r^4 h^2$$

$$z = \frac{1}{9}\pi^2 r^4 (\ell^2 - r^2)$$

$$z = \frac{1}{9}\pi^2 r^4\left[\left(\frac{S}{\pi r} - r\right)^2 - r^2\right]$$

$$\frac{dz}{dr} = 0 \Rightarrow r = 0 \text{ or } r^2 = \frac{S}{4\pi}$$

$$\frac{d^2z}{dr^2}\Big|_{r^2=\frac{S}{4\pi}} < 0$$

$4\pi r^2 = S = \pi r\,(r + \ell)$

$3\pi r^2 = \pi r\ell$

$$\frac{r}{\ell} = \frac{1}{3} \text{ or } \sin\alpha = \frac{1}{3} \Rightarrow \alpha = \sin^{-1}\frac{1}{3}$$

Sol.8 $$\frac{dy}{dx} = \frac{12x}{(x^2+3)^2} = g(x) \text{ (let)}$$

$g'(x) = 0$ at $x = \pm 1$

maximum slope

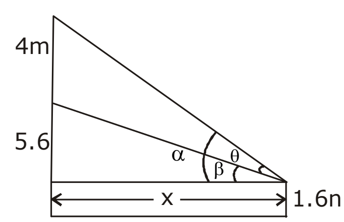
$g(1) = 3/4$ & point $(1, 3/2)$

minimum slope

$g(-1) = -3/4$ & point $(-1, 3/2)$.

Sol.9 from figure $\tan\theta = \tan(\alpha - \beta)$

$$\tan\theta = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$



put tan α & tan β

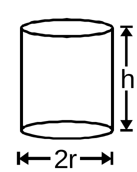
in terms of x & $\frac{d\theta}{dx}$.

Sol.10 Given : 2r + h = 3

volume

$v = \pi r^2 h$

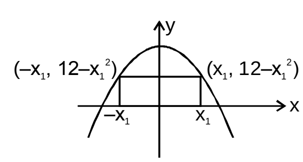
$= \pi r^2(3 - 2r)$



Sol.11 Area

$A = 2(12 - x_1^2).x_1$

$$\frac{dA}{dx_1} = 0 \Rightarrow x_1 = 2$$



A = 32 sq. units.

Sol.12 $x^2 h = 1000$

Top portion = x^2

Base = x^2

Sides = $4xh$

$E = 15x^2 + 25x^2 + 20(4xh) + 300$

$$E = 40x^2 + 80x\left(\frac{1000}{x^2}\right) + 300$$

$$E = 40x^2 + \frac{80000}{x} + 300$$

$$\frac{dE}{dx} = 0 \Rightarrow x = 10$$

$$\frac{d^2E}{dx^2} = 80 + \left.\frac{160000}{x^3}\right|_{x=10} > 0$$

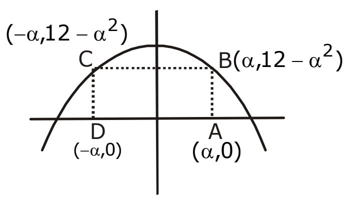
$\Rightarrow x = 10 \Rightarrow h = 10$

Sol.13 $A = 2\alpha(12 - \alpha^2)$

$$\frac{dA}{d\alpha} = 2(12 - 3\alpha^2) = 0$$

$\Rightarrow \alpha = \pm 2$

$$\frac{d^2A}{d\alpha^2} = -12\alpha\big|_{\alpha=2} < 0$$



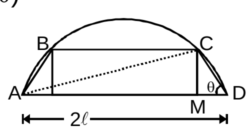
$A_{max} = 4(12 - 4) = 32$ sq. units

Sol.14 $CD = 2\ell\cos\theta$

$CM = 2\ell\cos^2\theta$

$P = 4\ell(1 + \cos\theta - \cos^2\theta)$

$$\frac{dp}{d\theta} = 0 \Rightarrow \theta = \frac{\pi}{3}, 0$$



$$\left.\frac{d^2p}{d\theta^2}\right|_{\pi/3} < 0 \quad \text{so} \quad \theta = \frac{\pi}{3}$$

Sol.15 $$\frac{dy}{dx} = \frac{(2x-5)(ax+b) - a(x^2 - 5x + 4)}{[(x-1)(x-4)]^2}$$

at x = 2 & –1

$$\frac{dy}{dx} = 0 \Rightarrow a = 1 \ \& \ b = 0$$

so Now $y = \dfrac{x}{x^2 - 5x + 4}$

Now find $\dfrac{dy}{dx}$ & check maximum value.

Sol.16 Let a, b, c be the sides and $a + b + c = 2s$

$\Delta^2 = s(s-a)(s-b)(s-c)$

by AM $\geq$ GM

$$\Rightarrow \frac{(s-a)+(s-b)+(s-c)}{3} \geq \sqrt[3]{(s-a)(s-b)(s-c)}$$

$$\therefore \frac{3s - (a+b+c)}{3} \geq \sqrt[3]{(s-a)(s-b)(s-c)}$$

$$\text{or } \left(\frac{s}{3}\right)^3 \geq (s-a)(s-b)(s-c)$$

$$\therefore \frac{s^4}{27} \geq s(s-a)(s-b)(s-c)$$

$$\therefore \frac{s^4}{27} \geq \Delta^2 \quad \Rightarrow \therefore \frac{s^2}{3\sqrt{3}} \geq \Delta.$$

$\therefore$ Δ has the maximum value $\dfrac{s^2}{3\sqrt{3}}$ and it takes place when AM = GM. hence a = b = c

Sol.17 $xy = 18$;

after marging ; $\ell = x - 3/4$, $w = y - 1/2$

so $A' = xy - \dfrac{x}{2} - \dfrac{3}{4}y + \dfrac{3}{8}$

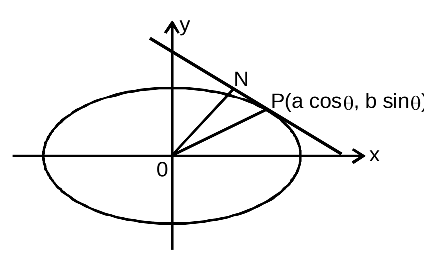
$$A' = 18 - \frac{x}{2} - \frac{3}{4}\cdot\frac{18}{x}$$

$$\frac{dA'}{dx} = 0 \Rightarrow x = 3\sqrt{3}$$

$$y = 2\sqrt{3}$$

Sol.18 Let $P = (a\cos\theta, b\sin\theta)$

$$T : \frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$



$$ON = \frac{ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}} \ \& \ P = \sqrt{OP^2 - ON^2}$$

$$NP^2 = D(\text{let}) = (a^2 + b^2) - u - \frac{a^2b^2}{u}$$

where $u = a^2 \sin^2\theta + b^2 \cos^2\theta$

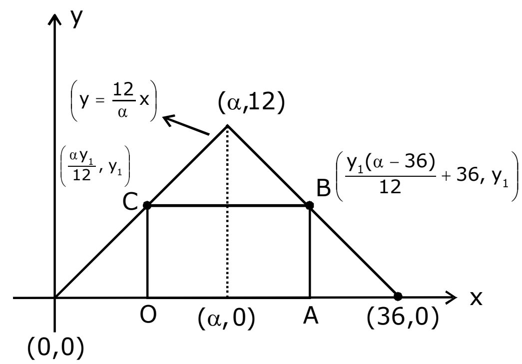
$$\frac{d(D)}{du} = -1 + \frac{a^2b^2}{u^2} \quad \& \quad \frac{d^2(D)}{du^2} = -\frac{2a^2b^2}{u^3}$$

$\downarrow$

$= 0 \Rightarrow u = ab \Rightarrow d'' < 0$

so D is maximum when $u = ab$

so $PN = |a - b|$



Sol.19 $y = \int_{-1}^{x} (t^2 - t)dt$

$$y = \frac{x^3}{3} - \frac{x^2}{2} + \frac{5}{6}$$

(a) $x = 0 \Rightarrow y = \frac{5}{6}$

$y = 0 \Rightarrow x = -1$

(b) $F'(x) = x^2 - x$

$F''(x) = 2x - 1$

(c) $F'(x) = 0 \Rightarrow x = 0, 1$

$\oplus$ $\ominus$ $\oplus$

0 1

$\uparrow$ ing in $(-\infty, 0) \cup (1, \infty)$

$\downarrow$ ing in $(0, 1)$

(d) $x = 0 \Rightarrow y = 5/6$

$x = 1 \Rightarrow y = 2/3$

(e) $f''(x) = 0 \Rightarrow x = \frac{1}{2}$ inflection point

Sol.21 Area of rectangle

$$= \left(\frac{y_1(\alpha - 36)}{12} + 36 - \frac{\alpha y_1}{12}\right).y_1$$

$A = 36y_1 - 3y_1^2$

$\frac{dA}{dy_1} = 0$ at $y_1 = 6$

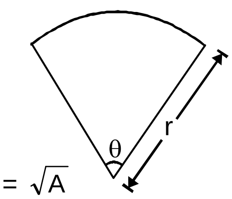
so for greatest area 6×18

Sol.22 $A = \pi r^2 \frac{\theta}{360}$

$p = r\theta + 2r$

put r in terms of A in P

& $\frac{dp}{dr} = 0 \Rightarrow r = \sqrt{A}$



Sol.23 $M(t) = \frac{3}{1 + 4e^{-t}}$

$\lim_{t \to -\infty} M(t) = 0$ & $\lim_{t \to \infty} M(t) = 3$

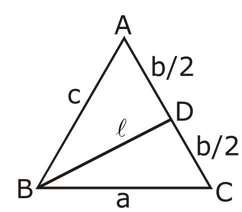
$$M = 3 - \frac{12}{e^t + 4}$$

$$\frac{dM}{dt} = \frac{-12e^t}{(et + 4)^2}$$

put in the given relation.

Sol.24 $b = c$

In ΔABD



$$\cos\theta = \frac{c^2 + b^2/4 - \ell^2}{bc}$$

$$\cos\theta = \frac{5b^2 - 4\ell^2}{4b^2} \quad ...(i)$$

$A = 2 \times \frac{1}{2} b^2 \sin\theta = b^2 \sin\theta$

$A^2 = b^4 \sin^2\theta$

$$= b^4 \left[1 - \left(\frac{5b^2 - 4\ell^2}{4b^2}\right)^2\right]$$

$$A^4 = b^4 \quad \frac{\left(5b^2 - 4\ell^2\right)^2}{16}$$

Diff. w.r.t. b
and put the value of b^2 in (i)

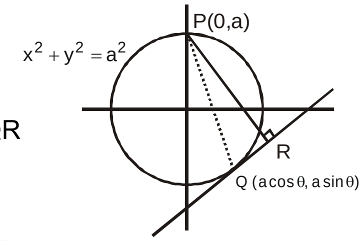
Sol.25 Tangent $x \cos \theta + y \sin \theta = a$

$PR = a(1 - \sin \theta)$

PQ = distance formula

Then by $PQ^2 = PR^2 + QR^2$

$QR = \sqrt{PQ^2 - PR^2}$



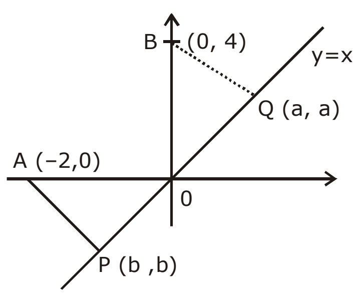
Area $= \frac{1}{2} \times PR \times QR$

$= f(\theta)$

$\frac{dA}{d\theta} = 0 \Rightarrow A = \frac{3\sqrt{3}}{8} a^2$

Sol.26 The co-ordinate M should lie in between P & Q.

Q is the foot of perpendicular from B to line y = x .



$\left(\frac{\alpha - 4}{\alpha}\right) = -1 \Rightarrow \alpha = 2$

$Q\ (2, 2) \Rightarrow BQ = 2\sqrt{2}$

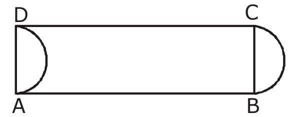
$AQ = 2\ \sqrt{5}$

$AB = \sqrt{20}\ = 2\ \sqrt{5}$

But perimeter is not the least in this case. So the co-ordinate of M will be (0, 0) for which perimeter is least.

Sol.27 V = volume of half cylinder

$V = \frac{1}{2}\ \pi r^2 h$



$S = s_1 + s_2 + s_3$

$s \rightarrow$ Total surface area

s_1 = semi circular surface

$= \frac{1}{2}\ \pi r^2 + \frac{1}{2}\ \pi r^2 = \pi r^2$

s_2 = surface of cylinder

$= \frac{1}{2}\ (2\pi rh) = \pi rh$

s_3 = Area of rectangular base whose height = h, width = 2r

$s_3 = 2rh$

$s = \pi r^2 + \pi rh + 2rh$

$s = \pi r^2 + (\pi + 2)\ r\ \frac{2v}{\pi r^2}$

$= \pi r^2 + \frac{2v}{\pi r}\ (\pi + 2)$

$\frac{dS}{dr} = 2\pi r - \frac{2v}{\pi r^2}\ (\pi + 2) = 0$

$\pi^2\ r^3 = V(\pi + 2)$

$\frac{d^2S}{dr^2} = 2\pi + \frac{4v(\pi + 2)}{\pi r^3}$

$2\pi + 4\pi = 6\pi > 0$

S least when

$\pi^2\ r^3 = V(p + 2)$

$\pi^2\ r^3 = \frac{1}{2}\ \pi r^2 h\ (x + 2)$

$\frac{h}{2r} = \frac{\pi}{\pi + 2}$

Sol.28 Let $f(x) = x^3 + 2Px^2 + P - a$

$f'(x) = 3x^2 + 4Px = 0$

$x = 0,\ x = -\frac{4P}{3}$

$f(0).\ f\left(-\frac{4P}{3}\right) < 0$

$(P - a)\left[-\frac{64}{27}P^3 + \frac{32}{9}P^3 + P - a\right] < 0$

$(P-a)\left[\frac{32}{27}P^3+P-a\right] < 0$

$(a-P)\left[a-P-\frac{32}{27}P^3\right] < 0$

If $P > 0$

$P < a < P+\frac{32}{27}P^3$

If $P < 0$

$\frac{32}{27}P^3+P < a < P$

Sol.29 $f(x) = e^{-ax} \cdot x^{a^2}$

$f'(x) = -a\, e^{-ax} \cdot x^{a^2} + a^2\, e^{-ax} \cdot x^{a^2-1} = 0$

$f'(x) = 0 \Rightarrow x = a$

min value at $x = a$

$F(a) = e^{-a^2} \cdot a^{a^2}$

$F(x) = e^{-x^2} \cdot x^{x^2}$

$F'(x) = -2x\, e^{-x^2} \cdot x^{x^2} + e^{-x^2}\, x^{x^2}\,(2x\,\ell n\, x + x) = 0$

$2x\,\ell n\, x - x = 0$

$x = 0,\ \ell n\, x = 1/2$

$x = \sqrt{e}$

min. value $= e^{-e}\left(\sqrt{e}\right)^e = e^{-e} \cdot e^{e/2} = e^{-e/2}$

Sol.30 $f(a) = \int_0^{1/a}(a^3+4x-a^5x^2)\,e^{ax}\,dx$

differentiate with respect to 'a' using leibnitz and equate to zero to get critical points and hence find minimum value.

Sol.31 **(a)** $f(0) = 0$

$\lim_{x\to0^+}\sqrt{x}\ \ell n\, x$

$= \lim_{x\to0^+}\frac{\ell n\, x}{x^{-1/2}}$ (By L'Hospital)

$= \lim_{x\to0^+}\frac{1/x}{-\frac{1}{2}x^{-3/2}} = 0$

f is continuous at $x = 0$

(b) $f'(x) = \frac{1}{2\sqrt{x}}\ \ell n\, x + \frac{1}{\sqrt{x}} = 0$

$\Rightarrow \ell n\, x + 2 = 0$

$\Rightarrow x = e^{-2}$

(c) $f'(0^+) = \lim_{h\to0}\frac{\sqrt{h}\,\ell n\, h - 0}{h}$

$\lim_{h\to0}\frac{\ell n\, h}{\sqrt{h}} \to \infty$

DNE

$\lim_{h\to0} f'(x)$ is also DNE

(d) $f'(x) = \frac{\ell n\, x + 2}{2\sqrt{x}}$

$f''(x) = \frac{2\sqrt{x}\left(\frac{1}{x}\right)-(\ell n\, x+2)\frac{1}{\sqrt{x}}}{4x}$

$f''(x) = 0 \quad \Rightarrow x = 0$

Sol.32 $f(x) = \ell n\,(1+\sin x)$

(a) zeroes of $f(x)$

where $\sin x = 0\ \forall\ x \in [-2\pi, 2\pi]$

$\Rightarrow x = -2\pi, -\pi, 0, \pi, 2\pi$

(b) $f'(x) = \frac{\cos x}{1+\sin x}$

$f''(x) = \frac{(1+\sin x)(-\sin x)-\cos^2 x}{(1+\sin x)^2}$

$= \frac{-\sin x - 1}{(1+\sin x)^2} \neq 0$

So, no inflection point.

(c) $f'(x) = 0$

$\Rightarrow x = \frac{\pi}{2}, -\frac{3\pi}{2}$

(d) Asymptotes where

$\sin x = -1$

$\Rightarrow x = -\frac{\pi}{2}, \frac{3\pi}{2}$

(e) $I = \int_{-\pi/2}^{\pi/2}\ell n(1+\sin x)dx$

king & add

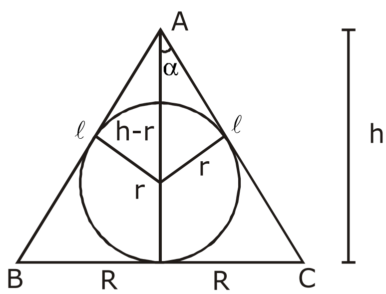
$$2I = \int_{-\pi/2}^{\pi/2} \ell n(1-\sin^2 x)dx$$

$$I = \int_0^{\pi/2} \ell n \cos^2 x \, dx = 2\int_0^{\pi/2} \ell n \cos x \, dx$$

$$= 2\int_0^{\pi/2} \ell n \sin x \, dx = 2\int_0^{\pi/2} \ell n \sin x \, dx$$

$I = -\pi \, \ell n \, 2$

Sol.33



$$\frac{r}{h-r} = \frac{R}{\ell} = \sin\alpha$$

$$\frac{r}{h-r} = \frac{R}{\sqrt{R^2+h^2}}$$

$r^2 (R^2 + h^2) = R^2 (h^2 - 2hr - r^2)$

$r^2 h^2 = R^2h^2 - 2hrR^2$

$r^2h^2 = R^2h (h - 2r)$

$$R^2h = \frac{r^2h^2}{h-2r}$$

$$V = \frac{1}{3}\pi R^2h$$

$$= \frac{1}{3}\pi\left(\frac{r^2h^2}{h-2r}\right) = \frac{1}{3}\pi\left(\frac{r^2}{\frac{1}{h}-\frac{2r}{h^2}}\right)$$

V should least $\Rightarrow z = \frac{1}{h} - \frac{2r}{h^2}$ (max.)

$$\frac{dz}{dh} = -\frac{1}{h^2} + \frac{4r}{h^3} = 0 \Rightarrow h = 4r$$

Sol.34 $f(x) = x^3 - \frac{3}{2}x^2 + \frac{5}{2} + \log_4 m$

$f'(x) = 3x^2 - 3x = 0$

$x = 0, 1$

$f(0) . f(1) < 0$

$$\left(\frac{5}{2}+\log_4 m\right)\left(1-\frac{3}{2}+\frac{5}{2}+\log_4 m\right) < 0$$

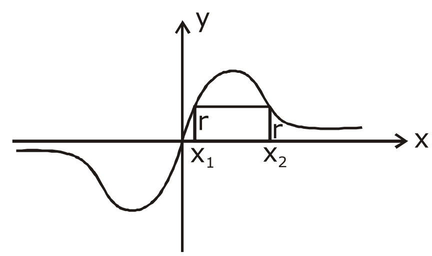
$$\left(\frac{5}{2}+\log_4 m\right)(2+\log_4 m) < 0$$

$$-\frac{5}{2} < \log_4 m < -2$$

$4^{-5/2} < m < 4^{-2}$

$$\frac{1}{32} < m < \frac{1}{16}$$

Sol.35 $\frac{dy}{dx} = 0$ at $x = \pm 1$



$$\frac{x_1}{x^2+1} = \frac{x_2}{x^2+1} \Rightarrow x_2 = \frac{1}{x_1}$$

so $U = \pi r^2 (x_2 - x_1)$

$$= \pi\left(\frac{x_2}{(1+x^2)^2}\right)\left(\frac{1}{x_1} - x_2\right) = \frac{\pi x_1(1-x^2)}{(1+x_1{}^2)^2}$$

$\frac{dy}{dx} = 0$ at $x = \sqrt{2} - 1$ is only accepted.

$U_{max.} = \pi/4$

Sol.36 $f'(x) = 3x^2 + 6(a-7)x + 3(a^2 - 9) = 0$

must have both roots positive

(i) $D > 0 \Rightarrow 36(a-7)^2 - 36(a^2-9) > 0 \Rightarrow a < \frac{29}{7}$

(ii) $f(0) > 0 \Rightarrow a \in (-\infty, -3) \cup (3, \infty)$

(iii) $-\frac{b}{2a} > 0 \Rightarrow \frac{-6(a-7)}{3} > 0 \Rightarrow a < 7$

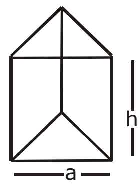
$\therefore\ a \in (-\infty, -3) \cup \left(3, \frac{29}{7}\right)$

$\Rightarrow\ a_2 = -3,\quad a_3 = 3,\quad a_4 = \frac{29}{7}$

$a_2 + 11a_3 + 70a_4 = 320$

Sol. 37 Given $s = 6a + 3h$ ....(i)

& volume $\Rightarrow$



$$v = \left(\frac{\sqrt{3}}{4}a^2\right).h$$

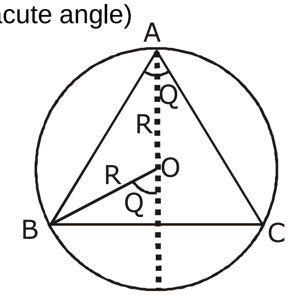
put h in (i) & $\frac{ds}{da} = 0\ \Rightarrow$ & get (a)

Sol. 38 $AB + BC + CA = L$

$$\frac{L = 2AB + BC}{L = 2R(1+\cos\theta)\sec\theta/2 + 2R\sin\theta/2}$$

$= 2R\ [2\cos\theta/2 + 2\sin\theta/2\cos\theta/2]$

$\frac{dZ}{d\theta} = 0$ at $\theta = \pi/3$ (acute angle)



$\theta \in \left(0, \frac{\pi}{2}\right)$

so at $\theta = 0$ & $\theta = \pi/3$ & $\theta = \pi/2$ check

& at $\theta = \theta$; $R = L/4$ (requred result)

Sol.39 $A = \frac{1}{2}BH$

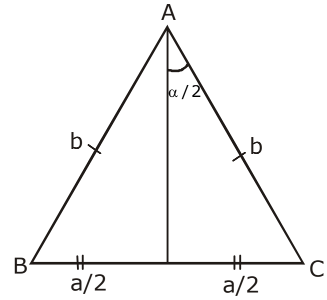
$$= \frac{1}{2}\left(b\cos\frac{\alpha}{2}\right)\left(2b\sin\frac{\alpha}{2}\right)$$

$$A = \frac{1}{2}b^2\sin\alpha$$

$$r = \frac{A}{s} = \frac{1}{2}\left(\frac{b^2\sin\alpha}{2b+a}\right)$$

$$r = \frac{1}{4}\left(\frac{b\sin\alpha}{1+a/2b}\right)$$

$$r = \frac{1}{4}\ \frac{b\sin\alpha}{1+\sin\frac{\alpha}{2}}$$



$$= \frac{1}{4}\sqrt{2A}\left(\frac{\sqrt{\sin\alpha}}{1+\sin\alpha/2}\right)$$

$$\frac{dr}{d\alpha} = 0 \quad \Rightarrow \cos\alpha = \sin\frac{\alpha}{2}$$

$$\frac{\pi}{2} - \alpha = \frac{\alpha}{2}\ \Rightarrow \alpha = \frac{\pi}{3}$$

Sol.40 $f'(x) = k(2x - x^2)e^{-x}$

(a) $f'(x) = 0 \Rightarrow x = 0, 2$

$\ominus$ 0 $\oplus$ 2 $\ominus$

$\uparrow$ ing in (0, 2)

$\downarrow$ ing in $(-\infty, 0) \cup (2, \infty)$

(b) $f''(x) = k(2 - 2x)e^{-x} - k(2x - x^2)e^{-x}$

$= ke^{-x}(2 - 2x - 2x + x^2)$

$= ke^{-x}(x^2 - 4x + 2)$

$f''(x) = 0$

$x^2 - 4x + 2 = 0 \Rightarrow x = 2 \pm \sqrt{2}$

concave up in $(-\infty, 2 - \sqrt{2}) \cup (2+\sqrt{2}, \infty)$

concave down $(2 - \sqrt{2}, 2 + 2\sqrt{2})$

(c) $f'(x) = k(2x - x^2)e^{-x}$

$f(x) = k\int (2x - x^2)e^{-x}\ dx$

$f(x) = k\ e^{-x}.\ x^2 + c$

$f(0) = 0 \Rightarrow c = 0$

$f(x) = k\ e^{-x}.\ x^2$

$f(2) = 2 \Rightarrow k = \frac{1}{2}e^2$

$f(x) = \frac{1}{2}x^2, e^{2-x}$

Sol.41 Let $f(x) = \sin x - \frac{2\pi}{\pi}$

$f'(x) = \cos x - \frac{2}{\pi}$

$f(x) \geq f(0)$

$f(x) \geq 0$

$\Rightarrow \sin x \geq \frac{2x}{\pi}$